



WORKSHEET

The Independent Events Question

Question Description: Question requires you to determine whether two events are independent. Following worksheets contains every time they have asked this style of question in the last 6 years.

Events A and B are **independent events** if:

$$P(A \text{ and } B) = P(A)P(B)$$

In English this means two events are independent if the outcome of one event does not affect the outcome of the other.

Events A and B are **Mutually exclusive** if:

$$P(A \text{ and } B) = 0$$

In English this means that two events are mutually exclusive if they cannot occur at the same time.

Example

Event A = "Watching Intuitive Videos"

Event B = "Getting an A* in Exam"

Are these events independent? No dude. If you watch Intuitive videos this will basically guarantee you an A*. If you don't watch well, you probably going to average like everyone else.

Are these events Mutually Exclusive? Are you trying to piss me off? It is impossible for these two events not to occur at the same time.

QUESTIONS

1

Jason throws two fair dice, each with faces numbered 1 to 6. Event A is ‘one of the numbers obtained is divisible by 3 and the other number is not divisible by 3’. Event B is ‘the product of the two numbers obtained is even’.

- (i) Determine whether events A and B are independent, showing your working. [5]
- (ii) Are events A and B mutually exclusive? Justify your answer. [1]

2

Ellie throws two fair tetrahedral dice, each with faces numbered 1, 2, 3 and 4. She notes the numbers on the faces that the dice land on. Event S is ‘the sum of the two numbers is 4’. Event T is ‘the product of the two numbers is an odd number’.

- (i) Determine whether events S and T are independent, showing your working. [5]
- (ii) Are events S and T exclusive? Justify your answer. [1]

3

Ashfaq throws two fair dice and notes the numbers obtained. R is the event ‘The product of the two numbers is 12’. T is the event ‘One of the numbers is odd and one of the numbers is even’. By finding appropriate probabilities, determine whether events R and T are independent. [5]

4

A fair six-sided die is thrown twice and the scores are noted. Event X is defined as ‘The total of the two scores is 4’. Event Y is defined as ‘The first score is 2 or 5’. Are events X and Y independent? Justify your answer. [4]

5

Two ordinary fair dice are thrown and the numbers obtained are noted. Event S is ‘The sum of the numbers is even’. Event T is ‘The sum of the numbers is either less than 6 or a multiple of 4 or both’. Showing your working, determine whether the events S and T are independent. [4]

6

In a group of 30 adults, 25 are right-handed and 8 wear spectacles. The number who are right-handed and do not wear spectacles is 19.

- (i) Copy and complete the following table to show the number of adults in each category. [2]

	Wears spectacles	Does not wear spectacles	Total
Right-handed			
Not right-handed			
Total			30

An adult is chosen at random from the group. Event X is 'the adult chosen is right-handed'; event Y is 'the adult chosen wears spectacles'.

- (ii) Determine whether X and Y are independent events, justifying your answer. [3]

7

For a group of 250 cars the numbers, classified by colour and country of manufacture, are shown in the table.

	Germany	Japan	Korea
Silver	40	26	34
White	32	22	26
Red	28	12	30

One car is selected at random from this group. Find the probability that the selected car is

- (i) a red or silver car manufactured in Korea, [1]
(ii) not manufactured in Japan. [1]

X is the event that the selected car is white. Y is the event that the selected car is manufactured in Germany.

- (iii) By using appropriate probabilities, determine whether events X and Y are independent. [5]

8

In a group of students, the numbers of boys and girls studying Art, Music and Drama are given in the following table. Each of these 160 students is studying exactly one of these subjects.

	Art	Music	Drama
Boys	24	40	32
Girls	15	12	37

- (i) Find the probability that a randomly chosen student is studying Music. [1]
- (ii) Determine whether the events ‘a randomly chosen student is a boy’ and ‘a randomly chosen student is studying Music’ are independent, justifying your answer. [2]

9

There are 300 students at a music college. All students play exactly one of the guitar, the piano or the flute. The numbers of male and female students that play each of the instruments are given in the following table.

	Guitar	Piano	Flute
Female students	62	35	43
Male students	78	40	42

- (i) Find the probability that a randomly chosen student at the college is a male who does not play the piano. [1]
- (ii) Determine whether the events ‘a randomly chosen student is male’ and ‘a randomly chosen student does not play the piano’ are independent, justifying your answer. [2]

ANSWERS

① (i) Events A & B are independent events if
$$P(A \cap B) = P(A) \times P(B)$$

Total possible
of rolls $= 6 \times 6 = 36 \quad \therefore \boxed{n = 36}$

Event A = one # is divisible by 3 (3, 6)
 { one # is NOT divisible by 3 (1, 2, 4, 5)

Event B = Product of both # is even.

Event A
possible rolls $= 31, 32, 34, 35, 61, 62, 64, 65$
 $13, 23, 34, 53, 16, 26, 46, 56$

Total = $n_A = 16$ possible rolls

$$\therefore P(A) = \frac{16}{36} = \frac{4}{9}$$

$$\boxed{P(A) = \frac{4}{9}}$$

Event B
possible Rolls = 12, 14, 16, 21, 22, 23, 24, 25,
26, 32, 34, 36, 41, 42, 43,
44, 45, 46, 52, 54, 56, 61
62, 63, 64, 65, 66

TOTAL = n_B = 27 possible rolls

$$\therefore P(B) = \frac{27}{36} = \frac{3}{4}$$

$$\boxed{P(B) = \frac{3}{4}}$$

Events A & B
occurring together = 32, 34, 61, 62, 64, 65
23, 43, 16, 26, 46, 56
(Rolls that occur
in both sets)

TOTAL = 12 possible rolls

$$\therefore P(A \cap B) = \frac{12}{36} = \frac{1}{3}$$

$$\therefore \boxed{P(A \cap B) = \frac{1}{3}}$$

$$P(A) \times P(B) = \left(\frac{4}{9}\right) \times \left(\frac{3}{4}\right) = \frac{12}{36} = \frac{1}{3}$$

$$\therefore P(A) \times P(B) = \frac{1}{3}$$

$$\therefore P(A \cap B) = P(A)P(B) = \frac{1}{3}$$

Thus events A & B are independent events.

(ii) Events A & B are mutually exclusive if:

$$P(A \cap B) = 0$$

$$\text{As } P(A \cap B) = \frac{1}{3} \neq 0$$

$\therefore$ Events A & B are NOT mutually exclusive

② (i)

$$\text{Total \# of Rolls} = 4 \times 4 = 16$$

$$\begin{array}{l} \text{Event S} \\ \text{possible} \\ \text{Rolls} \end{array} = 13, 31, 22 \quad (\text{Total} = 3)$$

$$\begin{array}{l} \text{Event T} \\ \text{possible} \\ \text{Rolls} \end{array} = 11, 13, 31, 33 \quad (\text{Total} = 4)$$

$$\therefore P(S) = \frac{3}{16}$$

$$\therefore P(T) = \frac{4}{16}$$

$$\begin{array}{l} \text{Events S \&frac{1}{2} T} \\ \text{occurring together} \\ \text{(possible Rolls)} \end{array} = 13, 31 \quad (\text{Total} = 2)$$

$$\therefore P(S \cap T) = \frac{2}{16}$$

$$P(S)P(T) = \left(\frac{3}{16}\right) \times \left(\frac{4}{16}\right) = \frac{3}{64}$$

$$\therefore P(S)P(T) = \frac{3}{64} \neq \frac{2}{16}$$

$$\therefore P(S \cap T) \neq P(S)P(T)$$

Two Events $T \nmid S$ are NOT independent

$$(ii) \text{ As } P(S \cap T) = \frac{3}{64} \neq 0$$

then Events $S \nmid T$ are not exclusive

③ Total possible rolls = $6 \times 6 = 36$

Event R possible rolls = 26, 62, 34, 43
(Total = 4)

Event T possible rolls = 12, 14, 16, 21, 23, 25, 32, 34, 36, 41, 43, 45, 52, 54, 56, 61, 63, 65
(Total = 18)

$$\therefore P(R) = \frac{4}{36} = \frac{1}{9} \quad \therefore P(T) = \frac{18}{36} = \frac{1}{2}$$

Events R & T occurring together possible rolls = 34, 43 (Total = 2)

$$\therefore P(R \cap T) = \frac{2}{36} = \frac{1}{18}$$

$$P(R)P(T) = \left(\frac{1}{9}\right)\left(\frac{1}{2}\right) = \frac{1}{18}$$

$$\therefore P(R \cap T) = P(R)P(T) = \frac{1}{18} \text{ thus}$$

Events R & T are independent

④ Total possible rolls = $6 \times 6 = 36$

Event X
possible rolls = 13, 22, 31 (Total = 3)

Event Y
possible rolls = 21, 22, 23, 24, 25, 26 (Total = 12)
31, 32, 33, 34, 35, 36

$$\therefore P(X) = \frac{3}{36} = \frac{1}{12} \quad P(Y) = \frac{12}{36} = \frac{1}{3}$$

Event X & Y
occurring together = 22 (Total = 1)
possible rolls

$$\therefore P(X \cap Y) = \frac{1}{36}$$

$$P(X) P(Y) = \left(\frac{1}{12}\right) \left(\frac{1}{3}\right) = \frac{1}{36}$$

$$\therefore P(X \cap Y) = P(X) P(Y) = \frac{1}{36}$$

$\therefore$ Events X & Y are independent.

⑤

$$\text{Total possible Rolls} = 6 \times 6 = 36$$

$$\begin{aligned} \text{Event S} &= 11, 13, 15, 22, 24, 26, 31, 33, 35, \\ \text{possible Rolls} &42, 44, 46, 51, 53, 55, 62, 64, 66 \\ &(\text{Total} = 18) \end{aligned}$$

$$\begin{aligned} \text{Event T} &\Rightarrow 11, 12, \underline{13}, 14, 21, \underline{22}, 23, \underline{31}, 32, 41 \\ \text{possible Rolls} &(\text{sum} < 6) \end{aligned}$$

$$\begin{aligned} &\Rightarrow \underline{13}, \underline{22}, 26, \underline{31}, 35, 44, 53, 62, 66 \\ &(\text{sum is multiple of 4}) \end{aligned}$$

$$\begin{aligned} \therefore \text{Event T} &= 11, 12, 13, 14, 21, 22, 23, 26, 31 \\ \text{possible Rolls} &32, 35, 41, 44, 53, 62, 66 \\ &(\text{Total} = 16) \end{aligned}$$

$$P(S) = \frac{18}{36} = \frac{1}{2} \quad P(T) = \frac{16}{36} = \frac{4}{9}$$

$$\begin{aligned} \text{Events } S \cap T &= 11, 13, 22, 26, 31, 35, 44 \\ \text{occurring together} &53, 62, 66 \\ \text{possible Rolls} & \end{aligned}$$

$$\therefore P(S \cap T) = \frac{10}{36} = \frac{5}{18}$$

$$P(S)P(T) = \left(\frac{1}{2}\right)\left(\frac{4}{9}\right) = \frac{2}{9} \neq \frac{5}{18}$$

$$\therefore P(S \cap T) \neq P(S)P(T) \text{ thus}$$

Events S & T are NOT independent

⑥ (i)

	Wears spectacles	Does not wear spectacles	Total
Right-handed	6	19	25
Not right-handed	2	3	5
Total	8	22	30

(ii) Event X = Adult is R+H

Event Y = Adult wears Specs

$$P(X) = \frac{25}{30} = \frac{5}{6}$$

$$P(Y) = \frac{8}{30} = \frac{4}{15}$$

Event $X \cap Y$
Occurring together = Adult is both R+H & wears specs

$$P(X \cap Y) = \frac{6}{30} = \frac{1}{5}$$

$$P(X)P(Y) = \left(\frac{5}{6}\right)\left(\frac{4}{15}\right) = \frac{2}{9} \neq \frac{1}{5}$$

$\therefore P(X \cap Y) \neq P(X)P(Y)$ thus

Events $X \cap Y$ are NOT independent.

$$\textcircled{7} \quad n = 250$$

$$\begin{aligned} \text{(i)} \quad P(\text{Red or Silver}) &= \frac{(40 + 26 + 34) + (28 + 12 + 30)}{250} \\ &= \frac{170}{250} = \frac{17}{25} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad P(\text{NOT JAPAN}) &= 1 - P(\text{JAPAN}) \\ &= 1 - \frac{26 + 22 + 12}{250} = \frac{19}{25} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \text{Event } X = \text{car is white} \quad \therefore P(X) &= \frac{32 + 22 + 26}{250} \\ &= \frac{8}{125} \end{aligned}$$

$$\begin{aligned} \text{Event } Y = \text{Manufactured in Germany} \quad \therefore P(Y) &= \frac{46 + 32 + 28}{250} \\ &= \frac{2}{5} \end{aligned}$$

Event $X \cap Y$ = white & manufactured in Germany

$$P(X \cap Y) = \frac{32}{250} = \frac{16}{125}$$

$$P(X)P(Y) = \left(\frac{2}{5}\right)\left(\frac{8}{25}\right) = \frac{16}{125}$$

$$\therefore P(X \cap Y) = P(X)P(Y) = \frac{16}{125}$$

Thus X & Y events are independent.

$$\textcircled{2} \quad n = 160$$

$$(i) \quad P(\text{Music}) = \frac{40 + 12}{160} = \frac{13}{40}$$

$$(ii) \quad P(\text{Boy}) = \frac{24 + 40 + 32}{160} = \frac{3}{5}$$

$$P(\text{Boy} \cap \text{Music}) = \frac{40}{160} = \frac{1}{4}$$

$$P(\text{Boy}) P(\text{Music}) = \left(\frac{13}{40}\right) \left(\frac{3}{5}\right) = \frac{39}{200} \neq \frac{1}{4}$$

$$\therefore P(\text{Boy} \cap \text{Music}) \neq P(\text{Boy}) P(\text{Music})$$

$\therefore$ Events are NOT independent

$$(9) \quad n = 300$$

$$(i) \quad P(\text{Male who doesn't play piano}) = \frac{78 + 42}{300} = \frac{2}{5}$$

$$(ii) \quad P(\text{Male}) = \frac{78 + 40 + 42}{300} = \frac{8}{15}$$

$$\begin{aligned} P(\text{Not piano}) &= 1 - P(\text{piano}) \\ &= 1 - \frac{35 + 40}{300} = \frac{3}{4} \end{aligned}$$

$$P(\text{Male} \cap \text{Not piano}) = \frac{2}{5}$$

$$P(\text{Male}) P(\text{Not piano}) = \left(\frac{8}{15}\right) \left(\frac{3}{4}\right) = \frac{2}{5}$$

$$\therefore P(\text{Male} \cap \text{Not piano}) = P(\text{Male}) P(\text{Not piano}) = \frac{2}{5}$$

$\therefore$ Events are independent.